

# Polynomial Properties based Electronic Security System

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## Abstract

The polynomial has the various characteristics. The degree of the polynomial, the convergence of the sequence, recursive conditions, derivative etc. are the some noteworthy domain of the study of any polynomial. This paper applies these properties for designing security schemes. Thus the Electronic Security System (ESS) came in the existence as the transformation of the said properties.

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## 1. Introduction.

The polynomial has the various characteristics. The degree of the polynomial, the convergence of the sequence, recursive conditions, derivative etc. are the some noteworthy domain of the study of any polynomial. This chapter lies with the real polynomials and the real functions.

In 1976, Diffie and Hellman [8] introduced key exchange protocol as the application of discrete logarithm problem. Various security schemes [1-15] are surveyed.

We review Bernstein Polynomial over the mapping of real and natural numbers. Next, this starts with the Bernstein Polynomial.

This is given as below:

$$P_{m,k}(x) = \binom{m}{k} x^k (1-x)^{m-k}.$$

Where,

$$x \in [0,1]$$

And

$$k = \{0,1,\dots,m\}.$$

**1.1.The Generalised Bernstein Polynomial:** The formula is generalised by introducing the two new real numbers  $\alpha, \beta$ ; given in below:

$$\left(P_m^{(\alpha,\beta)} f\right)(x) = \sum_{k=0}^m p_{m,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right).$$

**1.2.Bernstein Operator:** This is obtained under the followings condition:

If,

$$\alpha = 0,$$

$$\beta = 0.$$

Similarly, the Schurer Operator can be obtained, which is given in the next section.

**1.3.Schurer Operator:** The following choice generates this:

If  $\alpha = 0$ ,

$p \in N_0$  and the following choice:

$$m \rightarrow m + p,$$

$$p \rightarrow m - p.$$

The proposed result is presenting in the next section.

**1.4.The Proposed Bernstein Polynomial:** By introducing the natural number in the existed Bernstein Polynomial, we formulated the new result as below:

$$m_n = \left\{ \min \left\{ \begin{array}{l} 0, [\alpha]; \alpha \in R/Z \\ 0, \alpha, \alpha \in R \end{array} \right. \right\}$$

A theorem is given in below for the above.

**1.5. Theorem:** Let the positive number be  $m = f(\alpha)$ , then  $\beta = \alpha + p$ .

Proof: By contradiction,

$$\alpha = f(m),$$

Then,

$$\alpha = \left\{ \max \left\{ \begin{array}{l} 1, [m]; m \in R/Z \\ 1, 1 - m; m \in R \end{array} \right. \right\}$$

But,

$$m \rightarrow m + p,$$

And,

$$k = \{0, 1, \dots, m\}.$$

Thus,

$m = \{0, 1, \dots, k\}$ ; This is impossible, because;

$$m \in N.$$

But,

The resultant is the set.

The assumption is wrong.

Hence

$$\beta = f(\alpha, p),$$

Or,

$$\beta = \max f(\alpha),$$

Or,

$$\beta = \alpha + p.$$

This completes the proof.

Next, two lemmas we prove.

**1.6. Lemma:** There is a rational expression of  $(\alpha, \beta, k, m)$  in  $[\alpha, \beta]$ .

**Proof:** Since,

$$\alpha \geq 0,$$

$$\beta \in R.$$

Let,

$$x \in [0, 1],$$

$$k = \{0, 1, \dots, m\}.$$

Then,

$$m + \beta \geq \gamma_\beta; \forall m, m_0 \in N,$$

Where,

$$\gamma_\beta = m_0 + \beta,$$

By  $k$ -set,

$$k + \alpha = (m + \beta)\gamma_\beta,$$

$$\gamma_\beta = \frac{k + \alpha}{m + \beta}.$$

Therefore,

$$0 \leq \frac{k + \alpha}{m + \beta} \leq \mu^{(\alpha, \beta)}.$$

This completes the proof.

Next, another lemma will be proved on the boundedness of the two numbers  $(\alpha, \beta)$ .

**1.7. Lemma:**  $1 \leq \mu^{(\alpha, \beta)}$ .

**Proof:** Case 1: If,

$$\alpha \leq \beta,$$

Then,

$$\mu^{(\alpha, \beta)} = 1.$$

Case 2:

If,

$$\alpha > \beta,$$

Then,

$$\mu^{(\alpha, \beta)} = 1 + \frac{\alpha - \beta}{\gamma_\beta}.$$

Since,

$$\frac{k + \alpha}{m + \beta} \leq 1,$$

$$k + \alpha \leq m + \beta.$$

$$\Rightarrow k \neq m,$$

$$\alpha \neq \beta,$$

But,

$$\frac{\alpha - \beta}{\gamma_\beta} + 1 = \mu^{(\alpha, \beta)}.$$

Hence,

$$1 \leq \mu^{(\alpha, \beta)}.$$

This completes the proof.

The construction of the number  $m$  is depending on the boundedness of the set real numbers over the set of integers. But here we have seen that the number  $\beta$  is based on not only another number  $\alpha$  with respect to map with the number  $p$ . This result shows that if the positive operator select as  $\mu^{(\alpha, \beta)}$ , then the new function can be generated with both the integers, i.e.  $k$  and  $m$ .

## 2. Proposed ESS.

### 2.1. ESS.

The document be  $D$ ,

Then, by the above theorem (5.4 & 5.5.);

$$Dm_n = \left\{ \min \left\{ \begin{array}{l} 0, [\alpha]; \alpha \in R/Z \\ 0, \alpha, \alpha \in R \end{array} \right. \right\}$$

By theorem 5.5.;

The key is,

$$\beta = \alpha + p.$$

By Lemma 5.7.;

$$D \leq \mu^{(\alpha, \beta)}.$$

Then,

ESS deciphered by Lemma 5.6.;

$[\alpha, \beta]$ .

### 3. Conclusion.

The proposed ESS is based on the discrete curve over the Bernstein Polynomial. The Security of the system interacts with the boundedness of the the two variables. The document encrypts with these variables over the set minimized Bernstein's Polynomial.

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# Fuzzycryption

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## Abstract

Hard mathematical problem interacts with the security of any cryptography. Such mathematical problems are; factorization, discrete logarithm, lattice, quantum etc. In this paper, Hard Mathematical Problem (HMP) transform into the class comprises with the fuzzy called the Fuzzy Hard Problem (FHP). The message and the FHP generate a new system referred as Fuzzycryption.

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## 1. Introduction.

The message and the FHP will be treated as the two distinct sets and the fuzzy as the characterisation tool.

Although Zadeh [1] introduced the concept of fuzzy in 1965 but several applications are presented later. The phenomenon of fuzzy generalizes as the classified methodologies and then this applies for the computation. This paper presents a composition of fuzzy and word called fuzzy-word computation. Later Zadeh [2, 3] designed some new protocols. Here we establish this idea as FHP by the survey of [4-10] presented as below:

Let,

The original message be  $m$  represented as follows:

$$m = f(A) : A = \text{Alphabet},$$

As we know that, message is the set of words with the rule called grammar.

We define the grammar rule by the binary operation rule. As the number follows the binary rule, the same generalization will be applied on the words also.

The fuzzy cryptosystem is presented in below:

## 2. Fuzzy Encryption.

The mapping presents as below:

Let a word be,

$$w = f(l) : l = \text{letters}.$$

As the string, the word  $w$  is presented as,

$$w = w_1 w_2 \dots w_n.$$

The mapping of the binary operation with the string of two letters is presented in below:

$$+ \rightarrow ab \rightarrow a + b$$

$$- \rightarrow ba \rightarrow b - a$$

$$\times \rightarrow aa \rightarrow a \times b$$

$$\div \rightarrow ac \rightarrow c \div a$$

The illustration is given in below:

Let a word be “no”. It is performed as below:

$$no$$

$$\downarrow$$

$$n * o$$

$$\downarrow$$

$$n + l + m + n + o$$

The word i.e. the plaintext “no” became the ciphertext as " $n + l + m + n + o$ ".

The additional term is  $l + m + n$ .

Now, apply the fuzzy principle as:

The membership function: “+”.

The fuzzy set:  $(\{n, o\}, n + l + m + n + o)$ .

There will be unique " $l + m + n$ " for the word “no”.

How we decrypt the ciphertext, the next subsection lies with this:

### 3. Fuzzy Decryption.

The received message or the ciphertext is,

$$"n + l + m + n + o".$$

The applied algebraic operation will be shifted by the fuzzy operation as below:

Let A be the applied membership function as:

$$A : n * o.$$

B is another membership function as:

$$B : l + m + n.$$

The fuzzy operation for these two membership functions is denoted as:

$$f_{AB} = f_A \cdot f_B.$$

$$f_{A+B} = f_A + f_B$$

.

$$f_{A-B} = f_A - f_B$$

$$f_{A/B} = f_A / f_B.$$

Hence the fuzzy decryption will be proceeded as the inverse fuzzy operation of addition i.e. subtraction.

The ciphertext holds the following condition:

$$\text{The original message} = f_B - f_A.$$

#### 4. Conclusion.

Although cryptography started for the security of the message only in ancient but now it has generalized the digital society. Message transformation has become a science today. Recently biological aspect of the cryptography has launched. The natural secret also lies with this domain.

Hence the current work will be directed the future plan towards the security. This study is presented as the application of the fuzzy set and cryptography both. Zadeh's contribution was the landmark in the development of real mathematical application but today this has been applied in almost every field of the society.

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# New Data Structure Technique based on Artificial Neural Network

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## Abstract

New data structure technique is proposed in this paper by using the mechanism of Artificial Neural Network (ANN). The finite dimension of the field is generalized in this paper. The linear transformation and the representation of the vector basis are applied to propose the said structure.

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**1.Introduction:**

In 1977, Akaike et al [1] presented an advancement and an application of information theory to design data interpretation technique. In 1993, there are three research papers [2-3] are published on physiological application. This paper presents the medical data and its transformation as the resultant of the survey of [4-13] over the Jordon canonical form into the artificial neural network as below; A function  $\phi$  is defined on n-data of k-dimension of the matrix A. A standard basis is presented as below by the function  $\phi$  over the  $K_n$  field :

$$\phi : K^n \longrightarrow K^n$$

Let N be a set of neural network defined as :

$$N = \{ n_1, n_2, \dots, n_m \}$$

Then , the function  $\phi$  is defined as the linear transformation by

$$\phi : N \longrightarrow N$$

Let the field be k . the finite dimensional vector space N is defined over K.

Let , the polynomial be q defined as the minimal polynomial over the  $\phi$  – cyclic space .

Then,

$$q = n^r + a_{r-1} n^{r-1} + \dots + a_0 \in K \quad n$$

Where , n is defined as the sequence of neural network as ;



$$n = (n_1, n_2, \dots, n_m)$$

and  $r$  is defined as the dimension of the field  $k$  as :

$$r = \dim_k N$$

Hence,  $N$  has an ordered basis  $V$  relative to which the matrix of  $\phi$  defined by:

$$A = \begin{bmatrix} 0 & 1_k & & & 0 \\ & & \ddots & & \\ & & & 1_k & \\ -a_0 & -a_1 & & & -a_{r-1} \end{bmatrix}$$

**1.1. The Representation:**

Then, the vector basis  $V$  is represented by ;

$$V = \left[ v, \phi(v), \phi^2(v), \dots, \phi^{r-1}(v) \right]$$

Where,  $v \in N$  .

This matrix is called a companion matrix of the monic polynomial as ;

$$Q \in N [ n ]^2$$

Such that ;

$$q = n + a_0$$

Then,

$$A = (-a_0)$$

By the neural network ; Another neural function is defined as ;

$$\Psi : N \longrightarrow N$$

This is a linear transformation of a finite dimensional vector space  $N$  over a field  $K$ .

Then ,  $N$  is a  $\Psi$  – cyclic space and  $\Psi$  has a minimal polynomial:

$$q = (n - \Psi)^r ;$$

$$b \in K ;$$

If and only if ;  $\dim_K E = r$  and  $N$  has an ordered basis relative to the function  $\Psi$  over the matrix  $B$  :

$$\begin{bmatrix} b & 1_k & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & b & 1_k & \cdot & \cdot & \cdot & \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & & & 1_k \\ 0 & \cdot & \cdot & \cdot & \cdot & & 0_b \end{bmatrix}$$

**1.2. The Proposed Data Structure:**

This  $r \times r$  matrix  $B$  is called the elementary Jordan matrix associated with the neural network function over the medical data  $M$ .

Thus ;

$$(n - b)^r \in k [n]$$

such that ;

$$r=1 \quad B=(b)$$

$$R \times R \times \dots \times R$$

If,  $R_1, R_2, \dots, R_n$  and  $C$  be modules over the neural network set  $N$  ;

$$N = \{ n_1, n_2, \dots, n_m \}$$

There is an identity .

Then a function ;

$$f : R \longrightarrow R$$

Or

$$f : R_1 \times \dots \times R_n \longrightarrow C$$

Is referred as R – multilinear represented by ;

$$F (b_1 , \dots , b_{i-1}rb + sb', b_{i+1} , \dots , b_n )$$

$$= rf ( b_1 , \dots , b_{i-1} , b_{i+1} , \dots , b_n ) + sf ( b_1 , \dots , b_{i-1} , b' , b_{i+1} , \dots , b_n )$$

Where ;

$$B \in R$$

And;

$$B = \{ b_1 , b_2 , \dots , b_n \}$$

Let ,

$$C = R,$$

Then, the new networks for b as;

$$B_1 , B_2 , \dots , B_n$$

represented by ;

$$B_1 = B_2 = \dots = B_n = B$$

There are two linear neural network as b and N defined as bilinear.

$$\text{So, } f : B^n \longrightarrow N^n$$

The symmetric is defined as ;

$$F(b_{\sigma 1}, \dots, b_{\sigma n}) = f(b_1, \dots, b_n)$$

There is the permutation  $\sigma \in N$

The skew – symmetric is defined as ;

$$f(b_{\sigma 1}, \dots, b_{\sigma n}) = (\text{sgn } \sigma) f(b_1, \dots, b_n)$$

The alternate network is defined with the function ;

$$F(b_1, \dots, b_n) = 0$$

Such that ;

$$b_i = b_j ; i \neq j$$

The elements of N is defined as the elements of matrix as ;

$$N = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \\ \vdots & & \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

Then the matrix d is defined as ;

$$d : B \times B \longrightarrow R$$

By

$$[(a_{11}, a_{12}) \mid (a_{21}, a_{22})] \longrightarrow a_{11}a_{22} \longrightarrow a_{12}a_{21}$$

Generalizing as ;

$$[(a_{11}, a_{12}, a_{13}), (a_{31}, a_{32}, a_{33})] \mid \longrightarrow a_{11}a_{12} a_{13}a_{31}a_{32} a_{33}$$

For  $n \times n$  matrices also with the distance determinant function  $f$ .

By slow – symmetric of the network with the medical data over the determinant function is defined by ;

$$0 = \text{Thusf}(b_1 + b_2, b_1 + b_2)$$

$$= f(b_1, b_1) + f(b_1 + b_2) + \dots + f(b_2, b_2)$$

$$= 0 + f(b_1, b_2) + f(b_2, b_1) + 0$$

Then,

$$(R^n)^n = R^n \oplus \dots \oplus R^n \text{ and}$$

$$(B^n)^n = B^n \oplus \dots \oplus B^n$$

Thus,

$$(R^n)^n \longrightarrow R$$

$$(B^n)^n \longrightarrow R$$

The efficient network is function with the R – Multi linear and B – Multi Linear by the followings;

$$F : (R^n)^n \longrightarrow R$$

And

$$F : (B^n)^n \longrightarrow R$$

Such That ;

$$F(n_1, n_2, \dots, n_m) = r$$

Where  $(n_1, n_2, n_3 \dots, n_m)$  is referred as the basis of the network N .

The error – corretor is set with the new set X;

$$X = \{ X_1, X_2, \dots, X_n \}$$

$$X \longrightarrow A$$

By

$$X_1 \longmapsto A$$

$$\begin{array}{ccc}
 X_2 & \xrightarrow{\quad} & A \\
 \cdot & & \\
 \cdot & & \\
 X_n & \xrightarrow{\quad} & A'
 \end{array}$$

The standard basis is;

$$\{e_1, \dots, e_n\}$$

Then,

$$(e_1, \dots, e_n) \xrightarrow{\quad} L_n$$

Thus, the uniqueness of the network is presented as ;

$$(X_1, \dots, X_n) \in (\mathbb{R}^n)^n$$

And

$$(X_1, \dots, X_n) \in (\mathbb{B}^n)^n$$

By Bilinear ;

$$(X_1, \dots, X_n) \in f : (\mathbb{R}^n)^n (\mathbb{B}^n)^n$$

So ,

$$X_i = (a_{i1}, \dots, a_{in})$$

$$= \sum_{j=1}^n a_{ij} e_j$$

By isomorphic property;

$$(\mathbb{R}^n)^n \cong f_n \mathbb{R} (X_1, \dots, X_n)$$



And

$$(B^n)^n \cong_{f_n} R (X_1, \dots, X_n)$$

Then

$$\begin{array}{ccc} (R^n)^n & \xrightarrow{\quad} & a_{ij} \\ (B^n)^n & \xrightarrow{\quad} & a_{ij} \end{array}$$

By multilinearity ;

$$\begin{aligned} F(x_1, \dots, X_n) &= f( (x + a)^n = \sum_{y_1} a_{1y_1} e_{y_1}, \sum_{y_2} a_{2y_2} e_{y_2}, \dots, \sum_{y_n} a_{ny_n} \\ &\quad e_{y_n}) \\ &= \sum_{j_1} \sum_{j_2} \dots \sum_{j_n} a_{1y_1} a_{2y_2} \dots a_{ny_n} f(e_{y_1}, \dots, e_{y_n}) \end{aligned}$$

Hence,

$$F(N_1, N_2, \dots, N_m) = \sum_{i=1} \sum_{y=1} a_{iy} n_{iy}$$

This ,

$$F(X_1, X_2, \dots, X_n) = 0$$

$$F(B_1, B_2, \dots, B_n) = 0$$

$$F(N_1, N_2, \dots, N_n) = 0$$

$$\text{if } (E_1, E_2, \dots, E_n) \xrightarrow{\quad} \text{In}$$

Hence ;  $f : R_n \longrightarrow R_n \quad | \longrightarrow N$

2. **Conclusion:** This data structure comprises with the finite mapping thus provides the semantic security. The medical data contains the multilinearity property thus by isomorphic mapping the data transforms into information in multi-layer basis.

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# New Cloud Domain

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## Abstract

The probabilistic application is presented in this paper. The new cloud domain is proposed over the discrete structure of the set by the probabilistic distribution. The stochastic process, distortion and permutation based methodology are applied to propose this new cloud model.

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**1. Introduction:**

Presnell et al [1] introduced MacMatch in 1993. This is a tool for pattern of prediction of the structure of the protein formation. Pritchard et al [2] developed neural net prediction model in 1994. This is performed for the patients of Alzheimer’s Patients.

Then similar works [2-10] are proposed as an application of the discrete structures. Reibnegger et al [11] gave the probabilistic model of diagnosis in 1991. The literatures [12-20] are studied to develop this cloud domain idea.

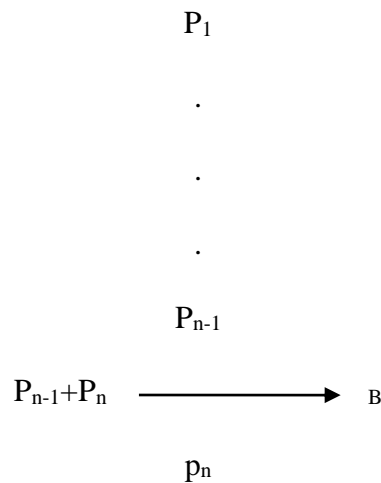
Next, the discrete probabilistic set is presenting.

**2. The Discrete Probabilistic Set:**

The structure of the information is defined by the set I;

$$I = \{ i_1, i_2, \dots, i_n \}$$

Thus the layer of the probability defined as;



Where B is the set of code words;

$$B = \{b_1, b_2, \dots, b_n\}$$

The efficiency interacts with the rate, say R;

$$R = \log_2 L \text{ bits}; L = X^2$$

or

$$R = [\log_2 L] + 1 \text{ bits}; L \neq x^2$$

where x is the representation of information.

There is the time set T defined by;

$$T = \{t_1, t_2, \dots, t_n\}$$

By stochastic process;

The message set X;

$$X = \{x_1, x_2, \dots, x_n\}$$

represented as;

$$X(t) = \{x_1(t_1), x_2(t_2), \dots, x_n(t_n)\}$$

By applying the probability; the information transmits as below;

This is a permutation;

$$X_1(t) = \{x_1(t_1), x_n(t_2), \dots, x_1(t_n)\}$$

$$X_2(t) = \{x_2(t_1), \dots, x_2(t_n)\}$$

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$$X_n(t) = \{x_n(t_1), \dots, x_n(t_1)\}$$

And

$$X(t_1) = \{x_1(t_1), \dots, x_n(t_1)\}$$

$$X(t_2) = \{x_2(t_2), \dots, x_2(t_2)\}$$

$$X(t_n) = \{x_n(t_1), \dots, x_n(t_n)\}$$

There is the error defined as;

$$d(x_1(t), x_2(t)) = (x_1(t) - x_2(t))^2$$

Similarly

$$d(x_1(t_1), x_2(t_2)) = (x_1(t_1) - x_2(t_2))^2$$

Hence, the average is measured by;

$$d(X_n(t_1), X_n(t_2)) = \frac{1}{n} \sum_{i=1}^n d(x_n(t_1), x_n(t_2))$$

This is called distortion.

Defined with the cloud set C as;

$$\begin{aligned} C &= E(d(X_1(T_1)), X_n(T_n)) \\ &= \frac{1}{n} \sum_{i=1}^n E[d(x_n(t_1)), x_n(t_n)] \end{aligned}$$



$$= E[d(x_n(t_1)), x_n(t_n)]$$

**3. The Proposed Cloud Domian:**

The variance in the cloud is defined by  $6x^2$  comprises with the distorsion cloud represented as;

$$R_g(\zeta) = \begin{cases} \frac{1}{2} \log_2 \left( \frac{6x^2}{c} \right) & ; 0 \leq c \leq 6x^2 \\ 0 & , c > 6x^2 \end{cases}$$

Hence the rate of distorsion cloud represented as:

$$C_g(R) = 2^{-2R} 6^2 x$$

Generalised to the probability  $P(X(t))$  as;

$$C_g(R) = 2^{-2R} 6^2 P(x(t))$$

**4. Cloud Matrix:**

Let the set of cloudsbe C, defined by;

$$C = \{c_1, c_2, \dots, c_n\}$$

This is represented by the generator polynomial  $g_{ij}(c)$  ;

$$g_{ij}(c) = \sum_l g_{ij}(c^l)$$

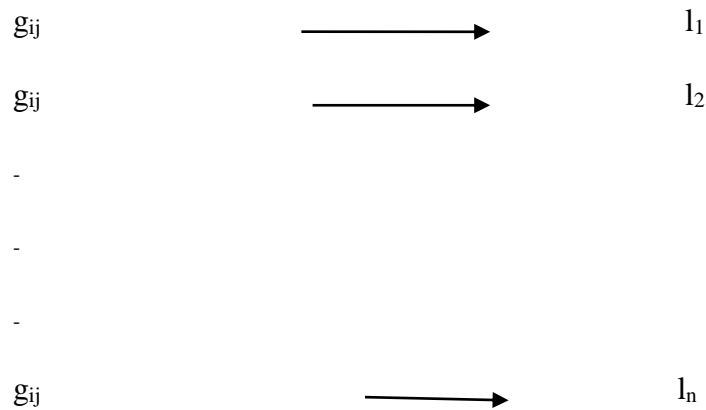
To find the generator matrix for each  $I$ ; there is the matrix represented by;

$$G_I$$

where  $g_{ij}$  corresponds to the  $I$  represented as;

$$G_I = [g_{ij}]$$

then the elements  $g_{ij}$  over the  $I$  are defined as;



where  $I$  is defined dynamically by;

$$(I_1, I_2, \dots, I_n)$$

Sequences;

Thus the matrix be  $G^{(n)}$ ; is represented as;

$$G = \begin{bmatrix} G_0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & G_n \\ 0 & G_0 & G_1 & G_2 & \dots & \dots & \dots & \dots & G_{n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & G_0 \end{bmatrix}$$

for the cloud measurement the matrix will be transformed into the convolution form represented as;

$$G = \begin{bmatrix} G_0 & G_1 & G_2 & G_n & 0 & 0 & 0 \\ 0 & G_0 & G_1 & G_{n-1} & G_n & 0 & 0 \\ \vdots & \vdots & G_0 & G_{n-2} & G_{n-1} & G_n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

This extends to the infinite terms for generating the new convolution form of G;

$$G = \left[ \begin{array}{c|c|c|c|c|c|c|c} I & P_0 & 0 & P_1 & \dots & \dots & \dots & 0 \\ 0 & 0 & I & P_0 & \dots & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \dots & \dots & \dots & \vdots \\ 0 & 0 & 0 & \dots & \dots & \dots & P_n & \\ \vdots & \vdots & \vdots & \dots & \dots & \dots & P_{n-1} & \\ \vdots & \vdots & \vdots & \dots & \dots & \dots & P_{n-2} & \end{array} \right]$$

Where

$p_0, p_1, \dots, p_n$  are the setting as parity check matrix

Next the distance (Metric) is defined on the set of clouds,

there is an upper bound be computed over the minimum distance of a convolutional code, rate depends on the elements of the channel  $C=(k_0, n_0)$

Then the constrain length is;

$$v = mk_0$$

and the rate of code is;  $n_0$

$$R = \frac{k_0}{n_0},$$

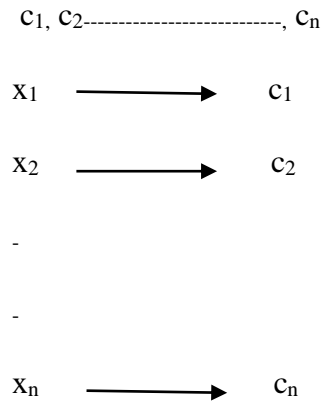
If the rate  $R$  and constraint length  $v$  there is the distance  $d$  defined as below:

$$H\left(\frac{d}{x_0 v}\right) \leq 1 - R$$

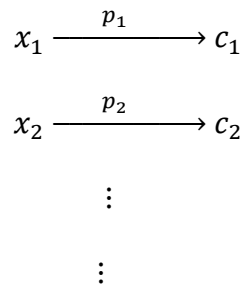
the entropy for this transmission is proposed as  $H(x)$ ;

$$H(x) = -x \log_2 x - (1-x) \log_2 (1-x); 0 \leq x \leq 1$$

the shifting by the element  $s$  of cloud;



By applying the probability over the above,



$$x_n \xrightarrow{p_n} c_n$$

Hence ,

$$d_1=(x_1,p(c_1))$$

$$d_2=(x_2,p(c_2))$$

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-

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$$d_n=(x_n,p(x_n))$$

and

$$d_1 \longrightarrow (c_1,p(c_1))$$

$$d_2 \longrightarrow (c_2,p(c_2))$$

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-

$$d_n \longrightarrow (c_n,p(c_n))$$

There is the probability defined discretely as below;

$$P=\{p_1,p_2-----p_n\}$$

Hence,

the optimized distance is defined by;

$$d_{\min}=(d_1,d_2)$$

$$\text{or } d_{\min}=(d_2,d_3)$$

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-

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$$d_{\min}=(d_{n-1},d_n)$$

By the discrete probability

$$d_{\min}=(d_1,p_1(d_1))$$

$$d_{\min}=(d_n,p_n(d_n))$$

The rate over this optimized distance  $s$  classified by:

$$c_1/p(d_1) \rightarrow p(c_1)$$

$$c_2/p(d_2) \rightarrow p(c_2)$$

⋮

⋮

$$c_n/p(d_n) \rightarrow p(c_n)$$

Thus the rate of transmission by cloud is defined as below;

$$\frac{p(c_1)}{d_1} ; \quad d_{min} \leq p(c_1)$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\frac{p(c_n)}{d_n} ; \quad d_{min} \leq p(c_n)$$

5. **Conclusion:** This cloud domain is the resultant of probabilistic transformation of distance. The stochastic process is applied on the finite set over the discrete probability distribution. The permutation of the data presented as the cloud structure in this paper. The proposed cloud domain provides the new discrete probabilistic set for further cloud domain.

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# An Extension of Fuzzy Basis and its Applications

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## Abstract

The fuzzy structure is analysed in this paper. The new structure is set as the optimized fuzzy class in the free abelian group. Some applications are also discussed in this paper. Network designing and security are the instant advantages of the proposed structure.

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**1. Introduction:** The grading and membership function comprises with the fuzzy. The fuzzy basis and its variants are applying for the developing of network protocols. The finiteness of this basis interacts with the efficiency and the rank with the security. Babuska et al [1] presented several algorithms based on fuzzy. Bandler et al [2, 3] also develops some operator for fuzzy implications in 1980. The literatures [3-8] are also based on fuzzy set and its applications. In 2000, Ruan et al [9] introduced another application of fuzzy logic by setting the new rules as If-Then rules. Later [10-19] also lies with the frontiers of fuzzy sets, fuzzy logic and its applications.

The proposed fuzzy basis and new variants are presented in next section.

## 2. The Proposed Fuzzy Basis:

Let,

F be defined for fuzzy classes in the free abelian group. The rank is finite i.e. n and G is a non zero subgroup of F , then, the fuzzy basis is ;

$$\{x_1, x_2, \dots, x_n\} \text{ of } F$$

There is an integer r ;

Such that ; ,

$$1 \leq r \leq n.$$

and the positive integer

Set D is defined as :

$$D = \{d_1, \dots, d_n\}$$

Such that

$$d_1 \mid d_2 \mid \dots \mid d_r$$

and

G is free abelian with the fuzzy basis ;

$$\{d_{1x_1}, \dots, d_{1x_1}\}$$

There is a Protocol to set the network by divisibility by ;

$d_1$  divides  $d_2$  and so on by

$$d_1 \mid d_2$$

$$d_2 \mid d_3$$

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$$d_{n-1} \mid d_n$$

Let,

$$n = 1,$$

Then ,

$$F = \langle n_1 \rangle \cong Z$$

and

$$G = \langle d_1 n_1 \rangle \cong Z$$

For

$$d_i \in N^*$$

Where  $N^*$  is the network over the operation  $*$ .

Thus,

The new basis is  $Y$ ;

of  $F$  and an element in  $G$  of the  $Y = \{y_1, \dots, y_n\} \cdot S$ , defined by

$$S y_1 + k_2 y_2 + \dots + k_n y_n$$

For:

$$k_i \in Z$$

The set  $Y$  ;

$$Y = \{y_1, \dots, y_n\}$$

For:  $i = 1, 2, \dots, n$ .

Hence, the set  $S$  ;

$$S = \emptyset$$

Where ,  $S$  has a least positive integer  $d_1$  for the particular basis;

$$\{y_1, y_2, \dots, y_n\} \in F$$

For  $v \in G$ .

Then

$$V = k_1 y_1 + k_2 y_2 + \dots + k_n y_n$$

By division algorithm ;

For the fuzzy-basis-network

On  $i=1, 2, \dots, n$ .

$$k_i = d_1 q_i + r_i$$

with;

$$0 \leq r_i < d_1$$

Then the fuzzy basis is represented by ,

Let , 
$$V = d_1(y_1 + q_2y_2 + \dots + q_ny_n) + r_2y_2 + \dots + r_ny_n .$$

Then , the protocol of the network on the fuzzy basis ;

$$W = \{x_1, y_2, \dots, y_n\} \quad x_1 = y_1 + q_2y_2 + \dots + q_ny_n ;$$

is a fuzzy basis.

Since ,

$$v \in G.$$

and,

$$r_i < d$$

Then,

W in any order is a basis of F , the optimal network on  $d_1$  in S.

This implies that ;

$$0 = r_2 = \dots = r_n$$

So that ;

$$d_1n_1 = v \in G.$$

Next, the parallel protocol will be H ;

$$H = \langle y_2, y_3, \dots, y_n \rangle$$

where ; H is also defined over the fuzzy basis and itself a free abelian group of rank n-1 such that ,



$$F = \langle x_1 \rangle \oplus H.$$

When, this protocol is defined over  $G$  and  $H$ .

Then ,

$$\langle V \rangle \oplus (G \cap H) = \langle d_1 x_1 \rangle \oplus (G \cap H).$$

Since ,

$\{ x_1, y_2, \dots, y_n \}$  is a basis of  $F$ ,

$$\langle V \rangle \cap (G \cup H) = 0.$$

If ,

$$u = t_1 x_1 + \dots + t_n x_n \in G$$

for

$$t_i \in G.$$

By the division algorithm ;

$$t_1 = d_1 q_1 + r_1$$

with

$$0 \leq r_1 < d_1$$

Thus ;

$$G : u - q_1 V.$$

$$\Rightarrow G = r_1 x_1 + \dots + t_n y_n.$$

The minimality of  $d_1$  in  $S$ , implies that ,

$$r_1 = 0.$$

Where ;

$$t_2 y_2 + \dots + t_n y_n \in G \cap H.$$

and

with

$$u = q_1v + r_1$$

$$0 \leq r_1 < d_1 .$$

∴

$$G = \langle V \rangle + (G \cap H)$$

The network over fuzzy basis is presented by ;

$$G = \langle d_1x_1 \rangle .$$

for ;

$$G \cap H \neq 0 .$$

Next is the protocol for the fuzzy network over the elements ;

$$\{x_2, \dots, x_n\} \text{ of } H .$$

There is the positive integers

$$r, d_2, d_3, \dots, d_r ;$$

such that ;

$$d_2 \mid d_3 \dots \mid d_r$$

and

$G \cap H$  is free – fuzzy abelian ,

The basis ;

$$\{ d_2x_2, \dots, d_rx_r \}$$

Since ;

$$F = \langle x_1 \rangle \oplus H$$

and

$$G = \langle d_1x_1 \rangle \oplus (G \cap H),$$

$$F = \{ x_1, x_2, \dots, x_n \}$$

and

$$G = \{ d_1x_1, \dots, d_r x_r \}$$

The network is followed by the division principle ;

$$d_1 \mid d_2.$$

and

$$d_2 = qd_1 + r_0.$$

with ;

$$0 \leq r_0 < d_1.$$

Hence ;

$$\{ x_2, x + qx_2, \dots, x_n \}:$$

is the basis of F.

Hence;

$$r_0x_2 + d_1(x_1 + qx_2) = d_1x_1 + d_2x_2 \in G$$

The minimality of  $d_1$  in S.

This implies the ;

$$r_0 = 0$$

where ;

$$d_1 \mid d_2.$$

**3. Conclusison:** The fuzzy basis and its variants are set some new network models in this paper. The discreteness of the network model are classified by the fuzzy set and fuzzy logic. An efficient and secure network is the resultant of this study.

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